

Gusset — verification against NAFEMS benchmarks

06 October 2026 · Gusset 0.11.0 · solver revision 6cee2ef · Windows 11

Summary

Gusset was run on 6 of the standard NAFEMS linear-elastic benchmarks, each asked for the **Maximum** setting (5 passes). LE1, LE5, LE10 and FV32 are within **1.4%** of the published target. **LE6 is not: -3.1%** (the program stopped a pass early; one more halving of the cells gives -0.1%). **FV52 is not: -3.6%** in its bending modes (the target is a plate-theory value; an independent meshed model of the same solid is lower still).

BENCHMARK	QUANTITY	TARGET	GUSSET	DIFFERENCE	UNKNOWN	TIME
LE1 — Elliptic membrane	Tangential edge stress σ_{yy} at D (x = 2 m, y = 0)	92.7 MPa	93.02 MPa	+0.34%	204,291	40 s
LE5 — Z-section cantilever	Axial stress σ_{xx} at A (mid-surface, free edge of the flange, x = 2.5 m)	-108 MPa	-109.52 MPa	+1.41%	228,492	46 s
LE6 — Skew plate under pressure	Largest principal stress on the lower face at the centre E	0.802 MPa	0.78 MPa	-3.07%	12,237	10 s
LE10 — Thick plate under pressure	Direct stress σ_{yy} at D (x = 2 m, y = 0, top face)	-5.38 MPa	-5.32 MPa	-1.16%	2,248,191	507 s
FV32 — Cantilevered tapered membrane	Natural frequencies, modes 1–6 (worst)	44.6–391 Hz	44.6–391 Hz	-0.05%	183,177	21 s
FV52 — Simply supported thick square plate	Natural frequencies, modes 4–10 (worst)	45.9–206 Hz	44.2–204 Hz	-3.65%	149,769	13 s

The targets are those of *The Standard NAFEMS Benchmarks* (TNSB, rev. 3). Each benchmark was run through the program's normal route: a STEP file of the geometry is imported, fixtures and loads are put on its CAD faces, and the adaptive solver is run — nothing is meshed by hand and no setting is tuned to the answer. To repeat it: `python -m validation.nafems 5`, then `python -m validation.document`.

How the solver works, briefly

The part is not meshed. It is laid over a regular grid of quadratic brick cells; cells cut by the surface are integrated over the part of them that lies inside the material, and fixtures are enforced on the true surface. Accuracy is set by the number of adaptive passes: each pass halves the cell size where the program judges it is needed and solves again, so a result that has stopped changing between passes has converged. Slender members are also checked against cells of the next order on the same grid, which finds a member too coarse

to show an effect at all; one that fails the check is refined along its length and reported as not resolved. A pass whose cells could not be made finer is not run, and the program says so. The tables below show every pass, not only the last.

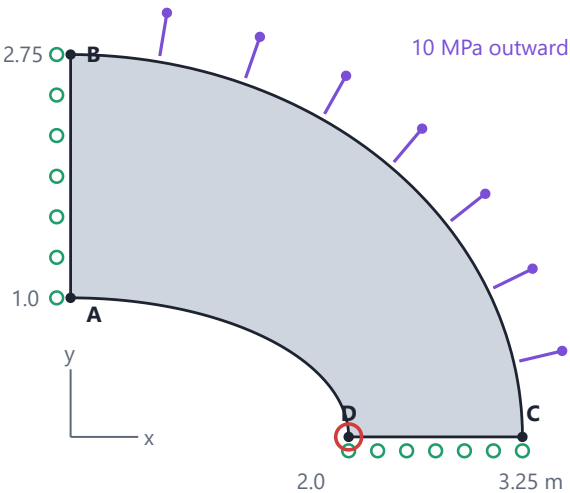
What these runs depend on

- **How finely curved faces are imported.** CAD faces are turned into flat facets before the solve. These runs used 128 facets per full circle; the program imports at 64. The stress at D is on the curved edge of the hole, where a coarse import leaves small corners that the finest cells begin to see.
- **How cut cells are integrated.** Cells cut by the surface are integrated on sub-cells, and where the surface runs through a sub-cell, exactly along rays that cross it: the material in each cell is then right to a few hundredths of a percent. Sampled at points instead, the stress read at a hole moved by several percent with the sampling.
- **Where the stress is read.** The value is the smoothed stress the program plots, taken 0.5 mm inside the surface. The unsmoothed value differs from it by at most 0.2% at the last pass of these benchmarks.
- **How the largest pass is solved.** A pass whose factors do not fit in memory (16 GB here) is solved iteratively over the grids of the passes before it, to a residual of 10^{-8} . Those passes are marked * in the tables (LE10).
- **The frequency benchmarks.** The program has no vibration analysis. FV32 and FV52 were run with a frequency solve written for this document, on the program's own stiffness and a mass matrix integrated over the same cut cells: they check the cells, the material in them and the supports, in a way a static stress at one point does not.
- **What is covered.** Six benchmarks: a membrane (in-plane) stress concentration, warping torsion of a thin-walled open section, thin-plate and thick-plate bending, and the natural frequencies of a membrane and of a thick plate; all single parts with rollers, held faces, pressure and end forces. They test the solver's cells, cut-cell integration, fixtures, refinement and stress recovery. They do not test welded or bolted joints between parts, or buckling; those, and beams and tubes in bending, are checked separately against meshed finite-element references and hand calculations in the program's test suite.
- **What was left out.** The other benchmarks of the linear-elastic set are thin shells of revolution (LE2, LE3, LE7–LE9), which a solid 3D method is not meant for, and a thermal load case (LE11), which the program does not have.

LE1 — Elliptic membrane

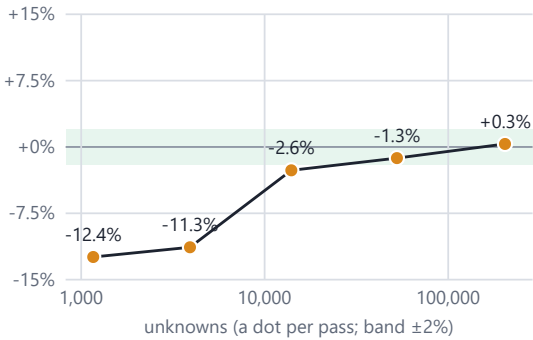
Plane stress in a plate with an elliptical hole, pulled outward round its outer edge. The stress concentrates at D, on the edge of the hole.

- Geometry
- Quarter of the region between the ellipses $(x/2)^2 + y^2 = 1$ and $(x/3.25)^2 + (y/2.75)^2 = 1$ m;
thickness 0.1 m
- Material
- $E = 210$ GPa, $\nu = 0.3$
- Loading
- Uniform outward pressure of 10 MPa on the outer edge BC
- Supports
- Edge DC: no movement in y. Edge AB: no movement in x (the two symmetry planes)
- Target
- Tangential edge stress σ_{yy} at D ($x = 2$ m, $y = 0$) =
92.7 MPa



As modelled. A solid plate 100 mm thick. The two symmetry edges are roller fixtures (normal direction held); one flat face is a roller too, which only stops the plate drifting through its thickness and leaves it free to thin, so the state is plane stress. The pressure is –10 MPa on the outer edge face. The stress is read 0.5 mm inside the material at D, because D lies on an edge of the surface.

PASS	SETTING	UNKNOWN	σ_{yy} AT D	DIFFERENCE	TIME
1	Draft	1,161	81.17	-12.44%	2.5 s
2	Fast	3,915	82.20	-11.33%	4.2 s
3	Standard	13,959	90.26	-2.64%	7.4 s
4	Accurate	52,533	91.53	-1.26%	15.8 s
5	Maximum	204,291	93.02	+0.34%	39.8 s



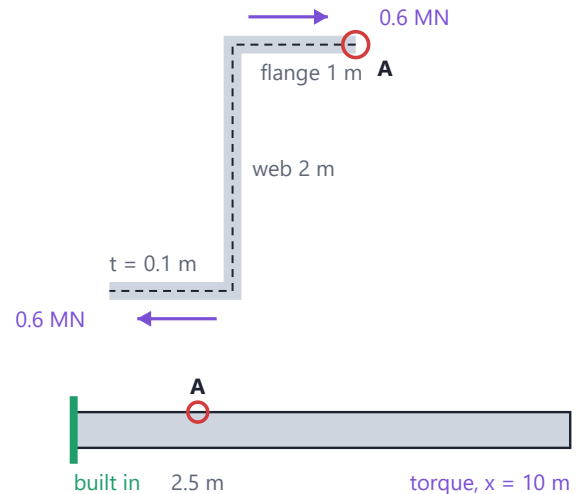
Times are cumulative from the start of the run.

The stress moves steadily toward the target as the cells shrink round the hole, and is within 3% at the Standard setting.

LE5 — Z-section cantilever

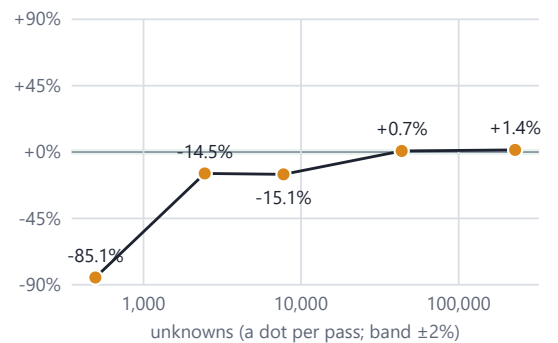
A thin-walled Z-section, built in at one end and twisted at the other. An open section resists torsion mostly by its flanges bending opposite ways (warping), which puts direct stress along the member; it is greatest at the free edges of the flanges.

Geometry	Length 10 m; web 2 m deep, flanges 1 m wide (mid-surface dimensions); thickness 0.1 m
Material	$E = 210 \text{ GPa}$, $\nu = 0.3$
Loading	A torque of 1.2 MN m at the free end, put on as a shear of 0.6 MN along the end of each flange, in opposite directions
Supports	All movement held at the end $x = 0$
Target	Axial stress σ_{xx} at A (mid-surface, free edge of the flange, $x = 2.5 \text{ m}$) = -108 MPa



As modelled. The benchmark is defined for shell elements. Here it is a solid, 100 mm thick about the mid-surfaces. The end face at $x = 0$ is a fixed fixture; each flange's end face carries a force of 0.6 MN across the member. The stress is read at mid-thickness, 0.5 mm in from the flange's free edge.

PASS	SETTING	UNKNOWN	σ_{xx} AT A	DIFFERENCE	TIME
1	Draft	495	-16.08	-85.11%	0.5 s
2	Fast	2,451	-92.38	-14.46%	1.4 s
3	Standard	7,731	-91.72	-15.07%	8.9 s
4	Accurate	43,527	-108.72	+0.67%	14.5 s
5	Maximum	228,492	-109.52	+1.41%	45.9 s



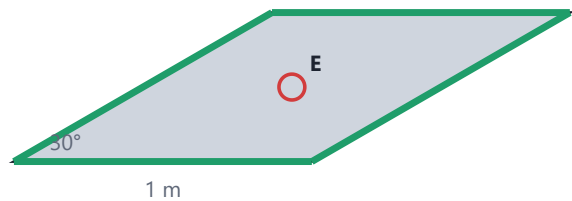
Times are cumulative from the start of the run.

The first passes are far off (-85.1%, -15.1% at the Standard setting): until the cells are about as small as the wall is thick they are too stiff for the flanges of an open section bending against each other. Halving the cells stalls on the way (two passes 15% off, the energy hardly moving between them), so the program does not go by that: after each pass it solves the member again with cells of the next order on the same grid, and while that changes more than 5% of the member's stress it refines the whole length (where memory has room for it) and reports the member as not resolved. That is what carries this case to the target at pass 4 (43,527 unknowns). At the Standard setting the result carries the note that the member is not resolved; open thin-walled sections in torsion need the Accurate setting.

LE6 — Skew plate under pressure

A thin rhombic plate, simply supported on its four edges and pressed on one face. The obtuse corners are singular, which makes the stress at the centre slow to settle.

- GeometryRhombus of side 1 m with corner angles of 30° and 150°; thickness 0.01 m
- Material $E = 210\text{ GPa}$, $\nu = 0.3$
- LoadingUniform pressure of 0.7 kPa on the top face
- SupportsNo deflection along the four edges
- TargetLargest principal stress on the lower face at the centre $E = \mathbf{0.802\text{ MPa}}$

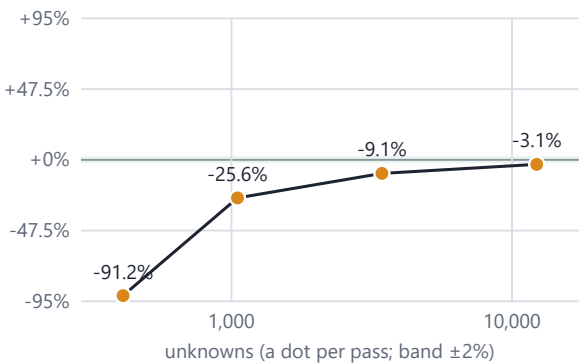


all four edges: no deflection
0.7 kPa on the face

As modelled. The benchmark is defined for shell elements. Here it is a solid 10 mm thick. The four edge faces are held in z only, which leaves them free to turn. To stop the plate drifting in its own plane it is held in x and y on two short lengths of one edge, at mid-thickness. (Holding a band of every edge in-plane, however narrow, clamps a plate this thin: the result was then 16% low.) The stress is read 0.05 mm inside the lower face.

PASS	SETTING	UNKNOWN	σ_1 AT E	DIFFERENCE	TIME
1	Draft	411	0.07	-91.21%	0.7 s
2	Fast	1,050	0.60	-25.65%	1.8 s
3	Standard	3,435	0.73	-9.07%	3.9 s
4	Accurate	12,237	0.78	-3.07%	9.6 s

Times are cumulative from the start of the run.

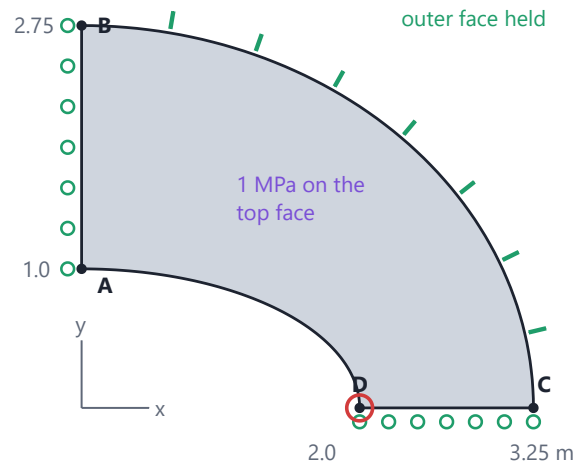


The stress rises steadily toward the target (–25.6%, –9.1%, –3.1%) and is –3.1% at the last pass the program ran. It stopped there: solved again with cells of the next order the plate's stress changed by 1.4% overall, under the 5% at which the program refines further, so it was taken as resolved. With the cells halved once more by hand the stress is within 0.1% of the target. The obtuse corners are singular, and the centre feels them: a check over the whole plate passes before the stress at one point has settled.

LE10 — Thick plate under pressure

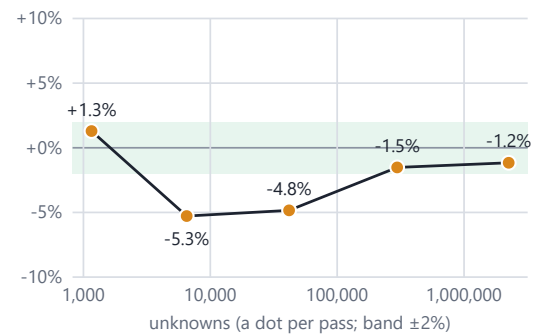
A thick elliptical plate with an elliptical hole, clamped in-plane round its outer edge and pressed on its top face: bending and shear through a thickness that is not small.

Geometry	The same plan form as LE1; thickness 0.6 m
Material	$E = 210 \text{ GPa}$, $\nu = 0.3$
Loading	Uniform pressure of 1 MPa on the top face
Supports	Face DCD'C': no movement in y. Face ABA'B': no movement in x. Outer face BCB'C': no movement in x or y, and none in z along its mid-height line EE'
Target	Direct stress σ_{yy} at D ($x = 2 \text{ m}$, $y = 0$, top face) = -5.38 MPa



As modelled. A solid plate 600 mm thick. The symmetry faces are roller fixtures; the outer face is held in x and y over its whole area and in z in a 12 mm band about mid-height (the line EE' of the definition, given a width the cells can see). The pressure is 1 MPa on the top face. The stress is read 0.5 mm inside the material at D, because D lies on an edge of the surface.

PASS	SETTING	UNKNOWN	σ_{yy} AT D	DIFFERENCE	TIME
1	Draft	1,161	-5.45	+1.29%	0.4 s
2	Fast	6,525	-5.10	-5.28%	1.4 s
3	Standard	41,877	-5.12	-4.84%	5.2 s
4	Accurate	297,687	-5.30	-1.51%	33.9 s
5	Maximum	2,248,191 *	-5.32	-1.16%	507.2 s



Times are cumulative from the start of the run.

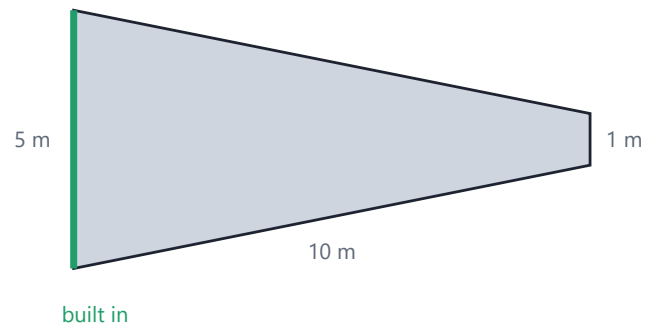
* solved iteratively: the factors of a direct solve do not fit in memory.

With only one or two cells through the plate the first passes are too coarse to mean much: the error grows from pass 1 to pass 2. From there it falls steadily (-5.3%, -4.8%, -1.5%, -1.2%). The pass-to-pass change the program reports is what shows that a coarse result has not settled.

FV32 — Cantilevered tapered membrane

A tapered plate built in at its wide end, vibrating in its own plane: bending and stretching modes of a deep, short cantilever.

Geometry	Length 10 m; 5 m deep at the root, 1 m at the tip; thickness 0.05 m
Material	$E = 200 \text{ GPa}$, $\nu = 0.3$, $\rho = 8000 \text{ kg/m}^3$
Loading	None (natural frequencies)
Supports	The root edge held in x and y; movement in the plane only
Target	Natural frequencies, modes 1–6 (worst): see the table



As modelled. A solid 50 mm thick. The root face is fixed; one flat face is on rollers, which keeps the motion in the plane and leaves the plate free to thin (plane stress).

MODE	TARGET	GUSSET	DIFFERENCE	PASS	UNKNOWN	WORST MODE	TIME
1	44.623 Hz	44.63 Hz	+0.01%	1	1,089	+0.60%	3.0 s
2	130.03 Hz	130.03 Hz	+0.00%	2	3,537	+0.06%	3.3 s
3	162.7 Hz	162.73 Hz	+0.02%	3	12,609	+0.04%	4.3 s
4	246.05 Hz	246.00 Hz	-0.02%	4	47,385	-0.04%	7.7 s
5	379.9 Hz	379.73 Hz	-0.05%	5	183,177	-0.05%	20.9 s
6	391.44 Hz	391.50 Hz	+0.01%				

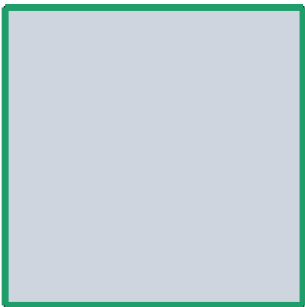
At pass 5, 183,177 unknowns.

Every frequency is within 0.05% of its target at the last pass, and within 0.6% on the first.

FV52 — Simply supported thick square plate

A thick square plate resting on the edges of its lower face: bending modes in which shear through the thickness matters, and in-plane modes.

- Geometry10 m × 10 m, 1 m thick
- Material $E = 200\text{ GPa}$, $\nu = 0.3$, $\rho = 8000\text{ kg/m}^3$
- LoadingNone (natural frequencies)
- SupportsNo deflection along the four edges of the lower face; free to slide and turn in its own plane (three rigid motions, left out)
- TargetNatural frequencies, modes 4–10 (worst): see the table



10 m, 1 m thick

lower edges: no deflection

As modelled. A solid. The lowest 50 mm of each side face is held in z — the line of the definition, given a height the cells can see. Because the plate is free to move as a body there is no static solution to steer the passes by: each pass halves the cells everywhere.

MODE	TARGET	GUSSET	DIFFERENCE	PASS	UNKNOWN	WORST MODE	TIME
4	45.897 Hz	44.22 Hz	-3.65%	1	729	-2.95%	0.1 s
5	109.44 Hz	107.42 Hz	-1.84%	2	2,601	-4.05%	0.4 s
6	109.44 Hz	107.42 Hz	-1.84%	3	9,801	-3.76%	1.2 s
7	167.89 Hz	163.13 Hz	-2.84%	4	38,025	-3.64%	4.3 s
8	193.59 Hz	193.60 Hz	+0.00%	5	149,769	-3.65%	12.9 s
9	206.19 Hz	203.66 Hz	-1.23%				
10	206.19 Hz	203.97 Hz	-1.08%				

At pass 5, 149,769 unknowns.

Not within 2%, and not expected to be. The in-plane mode (193.59 Hz), which the supports do not touch, is right to 0.00%. The bending modes are -3.6% to -1.1% and have settled there (the last two passes agree). The targets are those of thick-plate theory. A solid held along a line has no settled answer of its own: the finer the cells, the less a line holds. An independent conventional model of the same solid, held along the lines of the definition (`ref_fv52.py`: 27-node bricks, scikit-fem), gives 43.63 Hz for mode 4 (-4.9%) on $8 \times 8 \times 3$ elements and 43.21 Hz (-5.9%) on $16 \times 16 \times 4$, and the in-plane mode at 193.6 Hz. Held instead on the whole of each side face, the strictest reading of a simple support, this solver gives 44.44 Hz (-3.2%).